Portfolio Analysis and Optimization

A portfolio is made up of a number of securities. The relative value of each security is called the (relative) weight. The weight can be pre-determined by the owner or derived from mathematical optimization methods. A portfolio is just a weighted sum of individual components and can thus be analyzed as a single security.

Why do we need a portfolio?

Individual components of a portfolio can have different cyclical movements in prices and returns. One goal of a portfolio is for the movements to offset each other so that the composite return will be less volatile. The other goal is to seek high returns. The tradeoff between these two goals is the subject of portfolio analysis.

Consider a simple example of two securities:
VOO Vanguard S&P 500 ETF
VCLT Vanguard long-term corporate bond ETF

The inception date of VOO is 09/07/2010 whereas that of VCLT is 11/23/2009. VOO's data were first available on 09/09/2010. So all the analyses will be based on data from that day to our current date 09/21/2022.

Profile of component securities

By considering all one-year investment periods, we summarize the returns in the following.

1 Year Returns	VOO	VCLT	Explanation
Minimum	-20.28%	-29.41%	The minimum of all returns
First quartile	3.92%	-4.35%	One-fourth of returns are below this value
Median	13.08%	2.18%	The middle value
Third quartile	18.14%	9.16%	One-fourth are above this value
Maximum	77.25%	25.60%	The maximum return
Mean	12.33%	1.90%	The average
St Dev	11.61%	9.42%	The standard deviation

- As expected, the stock fund VOO has higher returns and wider spread.
- The bond fund has the lower returns, as reflected in the quartiles and the mean, and lower volatility.

So if one's goal is to have low volatility, then VCLT would be the choice. If the goal is high return, then VOO would be the choice. A portfolio can provide a balance of the two goals.

Sample Portfolio

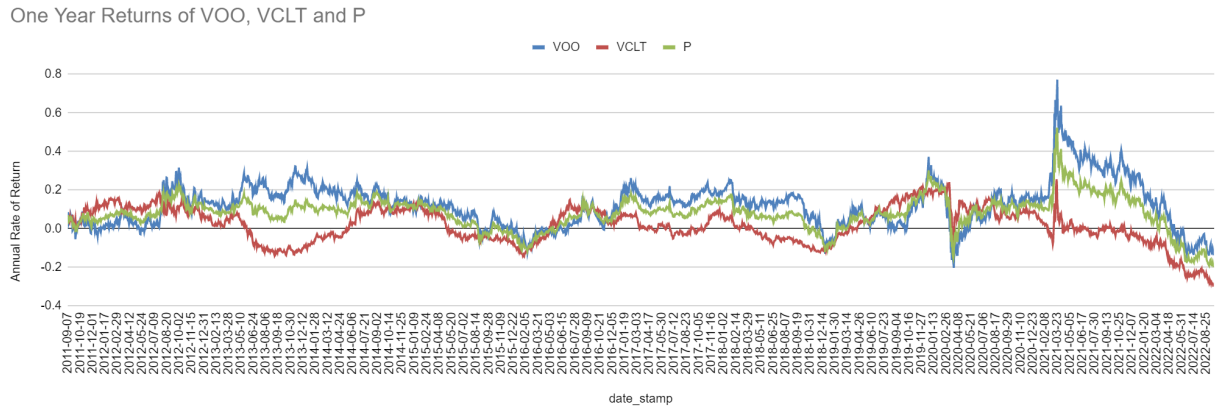
Following popular advice, suppose we allocate 60% of our money into VOO and the rest VCLT and call the result P.

1 Year	VOO	VCLT	P
Minimum	-20.28%	-29.41%	-19.97%
First quartile	3.92%	-4.35%	4.83%
Median	13.08%	2.18%	8.54%
Third quartile	18.14%	9.16%	12.56%
Maximum	77.25%	25.60%	51.86%
Mean	12.33%	1.90%	8.16%
St Dev	11.61%	9.42%	8.39%

- The standard deviation of P is even lower than that of the bond fund.
- The returns, as expected, are somewhere between those of the two funds. (The mean return is equal to the weighted sum of individual means.)

Time Chart

It is useful to look at the time series of the returns.

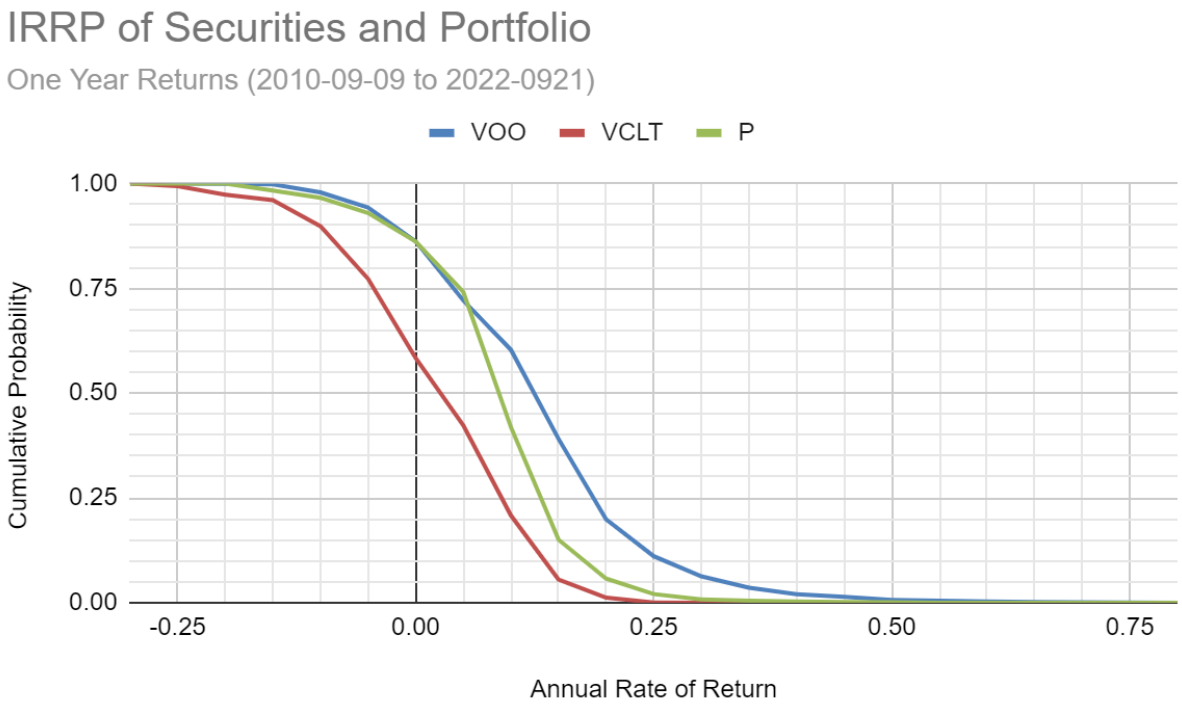


1. It can be seen that VOO and VCLT have different patterns of movement, with VOO being higher most of the time and with wider swings.
2. The coefficient of correlation between VOO and VCLT is 0.145, indicating the both move in the same up or down direction more often than not, but not to the same amplitude. Indeed there are periods when they move in opposite directions.
3. Portfolio P, being in the middle, dampens the swings of both funds; the effect is more visible when the funds move in opposite directions.

4. This graph shows the value of a portfolio; in that it smoothes the movements into a more stable series.

IRRP Graph

IRRP (Investment Return and Risk Profile) is our method of viewing the return and risk tradeoff using historical data. The method compiles all returns of a given investment duration into frequency distributions. The following is a cumulative relative frequency graph.



- The height of the curve is the (empirical) probability that the return is higher than the corresponding point on the x-axis. For example, at x=0.0, the height for both VOO and P is about 0.85. That means for both, the probability of making positive returns is 85%. That probability for VCLT is about 58%.
- Our measure of risk is the probability of not getting your desired return. So if your desired return is 5% or more, the graph shows that the risk of not getting it is about 25% with portfolio P, because the height of the curve at x=.005 is 0.75.
- The higher curve means higher probability of achieving the same return. So for almost the entire range of returns, VOO has the highest probability of success.
- The steeper the curve, the lower the volatility. From the graph, P has the steepest curve.

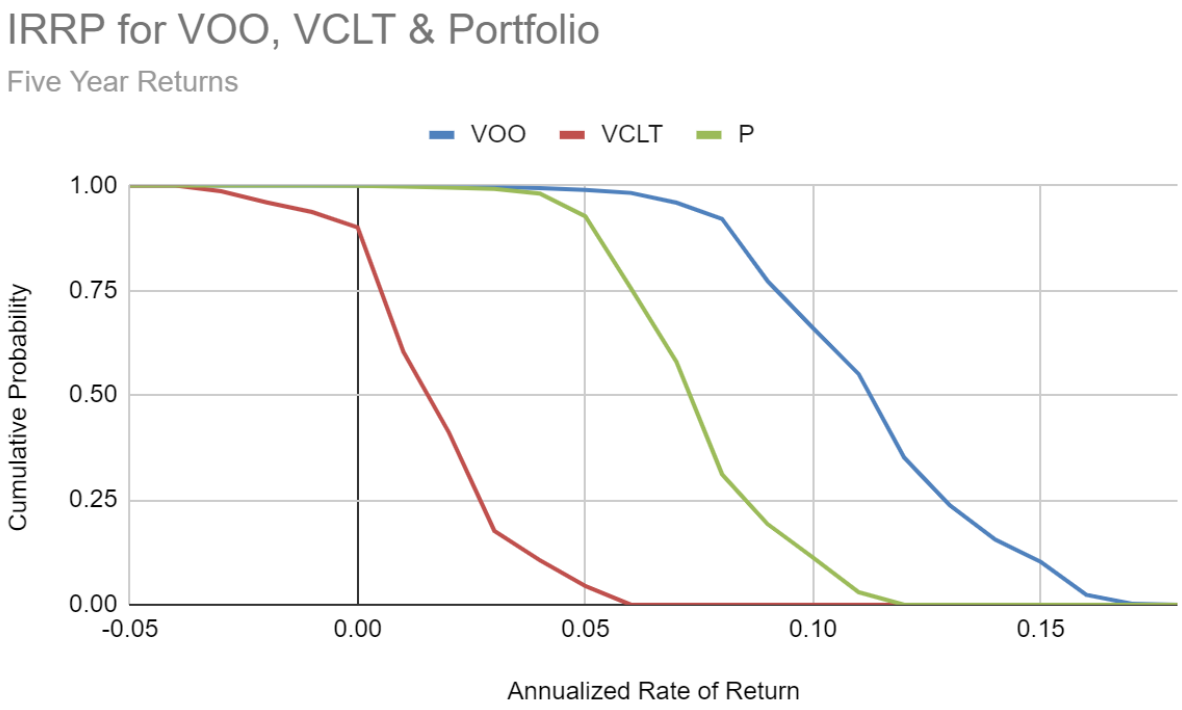
As we saw in the summary table, by combining VCLT with VOO, we lose some returns but gain lower volatility.

Using Five Year Returns

For long term investment, we recommend using returns from five or more years. The summary statistics for our sample portfolio are as follows.

5 Year	VOO	VCLT	P
Minimum	1.61%	-3.93%	-0.18%
First Quartile	9.18%	0.51%	6.04%
Median	11.30%	1.58%	7.22%
Third Quartile	12.88%	2.57%	8.41%
Maximum	17.41%	5.75%	11.84%
Mean	11.20%	1.62%	7.37%
St Dev	2.56%	1.77%	1.82%

The main differences with the one year returns are lower returns and much lower volatility in every fund and portfolio. This shows that time has its own dampening effect; therefore the effect of a portfolio is not as drastic.
The following graph shows that the portfolio is not as good as VOO, from the return and risk tradeoff point of view.



Portfolio Optimization

The Markowitz model of the so-called modern portfolio theory seeks to find the portfolio with the least volatility for a given target return. The model is based on annualized returns and a target average return. In the following, we show a number of example portfolios produced by the Markowitz model.

First, we use one-year returns and targets of 8, 9, and 10%.

One-Year Returns	.08	.09	.10
Weight(VOO,VCLT)	0.587, 0.413	0.682, 0.318	0.778, 0.222
St Dev	0.084	0.089	0.096
C.V.	1.05	0.99	0.96

- First observation is that as target increases, more weight is shifted to VOO since it offers greater returns.
- Increasing targets incur increased volatility.
- However, the increase in volatility is not as fast as the increase in target, as reflected by the coefficient of variation (CV) which is equal to standard deviation / mean.
- It's interesting to note that our previous example of 60-40 weight yields very close to the optimal standard deviation.

It may be instructional to compare the results using five-year returns.

Five-Year Returns	.08	.09	.10
Weight(VOO,VCLT)	0.666, 0.334	0.77, 0.23	0.874, 0.126
St Dev	0.019	0.021	0.023
C.V.	0.24	0.23	0.23

- For the same target returns, the weights shifted more to VOO.
- The volatility is much lower and the rate of increase, with respect to the mean, is also much lower.

Conclusions

With diverse securities, a portfolio can achieve desired returns with lower volatilities. For a long term investor, we recommend using 5 year or longer returns to evaluate the portfolio options. The optimized solutions can be used as a benchmark of one's own portfolio.